Level – 2 (Numerical Value Type)	Daily Tutorial Sheet - 12

141.(0) Put $\left(\frac{x}{e}\right)^x = t$ or $x\ln\left(\frac{x}{e}\right) = \ln t$ $\quad\therefore\quad (1+\ln x - \ln e)\,dx = \frac{1}{t}\,dt$

$\Rightarrow \quad \ln x\; dx = \frac{1}{t}\,dt \qquad \therefore \quad I = \int\left(t+\frac{1}{t}\right).\frac{1}{t}\,dt = \int dt + \int \frac{1}{t^2}\,dt = t - \frac{1}{t} + C$

$\therefore \quad I = \left(\frac{x}{e}\right)^x - \left(\frac{e}{x}\right)^x + C \qquad \therefore \qquad A = 1, \quad B = -1$

142.(1.25) $\frac{2x^2+1}{(x^4-4)(x^2-1)} = \frac{3}{x^2-4} - \frac{1}{x^2-1}$

$\therefore \quad I = \int \frac{3}{x^2-4} - \int \frac{1}{x^2-1}\,dx = \frac{3}{4}\ln\left|\frac{x-2}{x+2}\right| - \frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| + C = \ln\left[\left(\frac{x+1}{x-1}\right)^{\frac{1}{2}}\left(\frac{x-2}{x+2}\right)^{\frac{3}{4}}\right] + C$

$a = \frac{1}{2},\; b = \frac{3}{4}$

143.(8) $\int \sec^2 x\,\text{cosec}^4 x\;dx$

$= \int \frac{(\sin^2 x + \cos^2 x)^2}{\cos^2 x \sin^4 x}\,dx = \int \frac{\sin^4 x + \cos^4 x + 2\sin^2 x\cos^2 x}{\cos^2 x\sin^4 x}\,dx$

$= \int\left(\sec^2 x + 2\text{cosec}^2 x + \frac{\cos^2 x}{\sin^4 x}\right)dx = \tan x - 2\cot x + \int \cot^2 x\,\text{cosec}^2 x\;dx$

$= \tan x - 2\cot x - \frac{\cot^3 x}{3} + D$

144.(1.25) Let $\cos x = t$ or $\cos 2x = 2t^2 - 1$ and $dt = -\sin x\;dx$

$\therefore \quad I = \int \frac{t^2-2}{2t^2-1}\,dt = \frac{1}{2}\int\frac{2t^2-4}{2t^2-1}\,dt$

$= \frac{1}{2}\int dt - \frac{3}{2}\int\frac{dt}{2t^2-1} = \frac{1}{2}t - \frac{2}{4\sqrt{2}}\log\left|\frac{\sqrt{2}t-1}{\sqrt{2}t+1}\right| + C = \frac{1}{2}\cos x - \frac{3}{4\sqrt{2}}\log\left|\frac{\sqrt{2}\cos x - 1}{\sqrt{2}\cos x + 1}\right| + C$

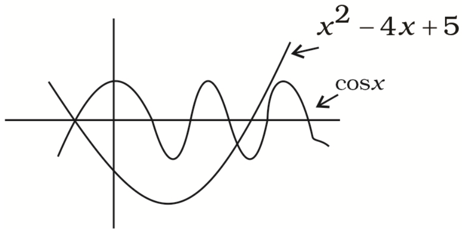
145.(2) $g(x) = -\cos\pi x + x^2 - 4x + C$

$g(1) = 3$

$\Rightarrow \quad 3 = 1 + 1 - 4 + C \quad\Rightarrow\quad C = 5$

$\therefore \quad g(x) = -\cos\pi x + x^2 - 4x + 5$

Exactly two solutions $\therefore\; k = 2$



146.(60) We have $f(x) = (x+1)^3$

Now, $\int f(x)dx = \int (x+1)^3 dx = \frac{(x+1)^4}{4} + C \;\Rightarrow\; g(x) = \frac{(x+1)^4}{4}$

Hence, $g(3) - g(1) = \frac{4^4}{4} - \frac{2^4}{4} = 64 - 4 = 60$

147.(4) $f'(x)=\dfrac{1}{1+\cos x}$; Integrating, $f(x)=\displaystyle\int\frac{dx}{2\cos^2\frac{x}{2}}=\frac{1}{2}\int\sec^2\frac{x}{2}dx=\frac{1}{2}.2.\tan\frac{x}{2}+C=\tan\frac{x}{2}+C$

$f(0)=3\Rightarrow C=3$; $f(x)=\tan\dfrac{x}{2}+3;\ f\left(\dfrac{\pi}{2}\right)=4$

148.(21) $I=\displaystyle\int\frac{dx}{x(x^{2007}+1)}=\int\frac{x^{2007}+1-x^{2007}}{x(x^{2007}+1)}dx=\int\left(\frac{1}{x}-\frac{x^{2006}}{1+x^{2007}}\right)dx$

$=\ln x-\dfrac{1}{2007}\ln(1+x^{2007})=\dfrac{\ln x^{2007}-\ln(1+x^{2007})}{2007}=\dfrac{1}{2007}\ln\left(\dfrac{x^{2007}}{1+x^{2007}}\right)+C$

$p+q+r=6021$

149.(22) As $Q(x)$ will be a polynomial of same degree as that of $P(x)$ and the leading coefficient of $Q(x)$ is equal to leading coefficient of $\dfrac{P(x)}{k}$

$\therefore\ \lim\limits_{x\to\infty}\dfrac{P(x)}{Q(x)}=\lim\limits_{x\to\infty}k=4$ Required = 7 + 7 + 4 + 4 = 22

150.(1) $\because\ \displaystyle\int\frac{x^{2009}}{1+x^{2010}}dx=\int\frac{x^{2010}}{x+x^{2011}}dx=\int\frac{1+x^{2010}}{x+x^{2011}}dx-\int\frac{1}{x+x^{2011}}dx$

$\Rightarrow\ \displaystyle\int\frac{x^{2009}}{1+x^{2010}}dx+\int\frac{1}{x+x^{2011}}dx=\int\frac{1+x^{2010}}{x(1+x^{2010})}dx\ \Rightarrow\ h(x)=\int\frac{dx}{x}=\ln x+C$

$\because\ h(1)=0\quad\because\ h(x)=\ln x\ \Rightarrow\ h(e)=\ln e=1$

151.(5) $\displaystyle\int\frac{2x+3}{(x^2+3x)(x^2+3x+2)+1}dx$

Put $x^2+3x=t\ \Rightarrow\ (2x+3)dx=dt$

$\displaystyle\int\frac{dt}{t(t+2)+1};\int\frac{dt}{(t+1)^2}=C-\frac{1}{t+1}=C-\frac{1}{x^2+3x+1}\ \Rightarrow\ a=1,b=3,c=1\Rightarrow a+b+c=5$

152.(40.3) $\displaystyle\int x(x^{2009}+x^{803}+x^{401})(2x^{1608}+5x^{402}+10)^{1/402}dx$

$\displaystyle\int\left(x^{2009}+x^{803}+x^{401}\right)\left(2x^{2010}+5x^{804}+10x^{402}\right)^{1/402}dx$

Put $2x^{2010}+5x^{804}+10x^{402}=t$; $4020(x^{2009}+x^{803}+x^{401})dx=dt$

$(x^{2009}+x^{803}+x^{401})dx=\dfrac{dt}{4020}\quad=\dfrac{1}{4020}\displaystyle\int t^{\frac{1}{402}}dt=\frac{1}{4020}\left(\frac{t^{\frac{1}{402}+1}}{\frac{1}{402}+1}\right)$

$=\dfrac{1}{4020}\left(\dfrac{t^{\frac{403}{402}}}{\frac{403}{402}}\right)=\dfrac{1}{4020}\times\dfrac{402}{403}t^{\frac{403}{402}}=\dfrac{1}{4030}(2x^{2010}+5x^{804}+10x^{402})^{\frac{403}{402}}$; $a=403$

153.(5) We have $\displaystyle\int\frac{1-7\cos^2 x}{\sin^7 x\cos^2 x}dx=\int\frac{\sec^2 x}{\sin^7 x}dx-7\int\frac{1}{\sin^7 x}dx=I_1-I_2$

Now, $I_1 = \int \underset{\text{(I)}}{\left(\frac{1}{\sin^7 x}\right)} \underset{\text{(II)}}{\sec^2 x}\, dx = \frac{\tan x}{\sin^7 x} + 7\int \frac{\tan x}{\sin^8 x}\cos x\, dx$

(By Parts) $= \frac{\tan x}{\sin^7 x} + I_2 \quad \therefore \quad I_1 - I_2 = \frac{\tan x}{\sin^7 x} + C$

Where C is a constant of integration

Hence, $g(x) = \tan x$

So, $g'(x) = \sec^2 x$ and $g''(x) = 2\sec^2 x \tan x \quad \therefore \quad g'(0) = 1$ and $g''\left(\frac{\pi}{4}\right) = 4$

Hence, $g'(0) + g''\left(\frac{\pi}{4}\right) = 1 + 4 = 5$

154.(12) $f(x) = \int \frac{1+x^4}{(1-x^4)^{3/2}}dx$

Take out x^2 from the Dr, it will come out as x^3

$$\int \frac{x + \frac{1}{x^3}}{\left(\frac{1}{x^2} - x^2\right)^{3/2}}dx$$

Put $\frac{1}{x^2} - x^2 = t^2 - 2\left(x + \frac{1}{x^3}\right)dx = 2t\, dt$

$f(x) = -\int \frac{t}{t^3}dt = \frac{1}{t} + C = \frac{x}{\sqrt{1-x^4}} + C_1$

As $f(0) = 0 \Rightarrow C_1 = 0$

Now, $\int \frac{x}{\sqrt{1-x^4}}dx = \frac{1}{2}\sin^{-1} x^2 + C_2$

But $g(0) = 0 \Rightarrow C_2 = 0$

$g(x) = \frac{1}{2}\sin^{-1} x^2$

Hence, $g\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{12} \equiv \frac{\pi}{k} \quad \Rightarrow \quad k = 12$

155.(2) $-1 \le \sin^2 x \le 0$

$\therefore \quad [-\sin^2 x] = -1 or\ 0 \quad \Rightarrow \quad \sec^{-1}[-\sin^2 x] = \pi$ as $\sec^{-1}(0)$ is not defined.

$\therefore \quad \int \sec^{-1}[-\sin^2 x]dx = \pi x + C$

$f(x) = \pi x$

$f\left(\frac{8}{\pi x}\right) = \pi \frac{8}{\pi x} = \frac{8}{x}$

$\left(f\left(\frac{8}{\pi x}\right)\right)'' = \frac{16}{x^3} \Rightarrow \left(f\left(\frac{8}{\pi x}\right)\right)''\Bigg|_{x=2} = \frac{16}{8} = 2$